Cambridge Assessment International Education
Cambridge International Advanced Level

MATHEMATICS **9709/32**

Paper 3 **May/June 2019**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2019 series for most Cambridge IGCSE™, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Question	Answer	Marks	Guidance
1	State unsimplified term in x^2, or its coefficient in the expansion of $(1+3x)^{\frac{1}{3}}$ $\left(\dfrac{\frac{1}{3}\times\frac{-2}{3}}{2}(3x)^2\right)$	**B1**	Symbolic binomial coefficients are not sufficient for the B marks
	State unsimplified term in x^3, or its coefficient in the expansion of $(1+3x)^{\frac{1}{3}}$ $\left(\dfrac{\frac{1}{3}\times\frac{-2}{3}\times\frac{-5}{3}}{6}(3x)^3\right)$	**B1**	
	Multiply by $(3-x)$ to give 2 terms in x^3, or their coefficients	**M1**	$\left(3\times\frac{10}{6}+1\right)$ Ignore errors in terms other than x^3 $3\times x^3\text{coeff} - x^2\text{coeff}$ and no other term in x^3
	Obtain answer 6	**A1**	Not $6x^3$
		4	

Question	Answer	Marks	Guidance
2	State or imply $u^2 - u - 12 (= 0)$, or equivalent in 3^x	B1	Need to be convinced they know $3^{2x} = (3^x)^2$
	Solve for u, or for 3^x, and obtain root 4	B1	
	Use a correct method to solve an equation of the form $3^x = a$ where a >0	M1	Need to see evidence of method. Do not penalise an attempt to use the negative root as well. e.g. $x \ln 3 = \ln a$, $x = \log_3 a$ If seen, accept solution of straight forward cases such as $3^x = 3$, $x = 1$ without working
	Obtain final answer $x = 1.26$ **only**	A1	The Q asks for 2 dp
		4	

Question	Answer	Marks	Guidance
3	Use correct trig formulae to obtain an equation in tan θ or equivalent (e.g all in sin θ or all in cos θ)	*M1	$\frac{1-\tan^2\theta}{2\tan\theta}=2\tan\theta$. Allow $\frac{\cot^2\theta-1}{2\cot\theta}=\frac{2}{\cot\theta}$
	Obtain a correct simplified equation	A1	$5\tan^2\theta=1$ or $\sin^2\theta=\frac{1}{6}$ or $\cos^2\theta=\frac{5}{6}$
	Solve for θ	DM1	Dependent on the first M1
	Obtain answer 24.1° (or 155.9°)	A1	One correct in range to at least 3 sf
	Obtain second answer	A1	**FT** $180°-\mathit{their}\ 24.1°$ and no others in range. Correct to at least 3 sf. Accept $156°$ but not 156.0 Ignore values outside range If working in $\tan\theta$ or $\cos\theta$ need to be considering both square roots to score the second A1 Mark 0.421, 2.72 as a MR, so A0A1
		5	

Question	Answer	Marks	Guidance
4	Use correct quotient rule	**M1**	Allow use of correct product rule on $x \times (1+\ln x)^{-1}$
	Obtain correct derivative in any form	**A1**	$\frac{dy}{dx} = \frac{(1+\ln x) - x \times \frac{1}{x}}{(1+\ln x)^2} = \left(\frac{1}{1+\ln x} - \frac{1}{(1+\ln x)^2}\right)$
	Equate derivative to $\frac{1}{4}$ and obtain a quadratic in ln x or (1+ ln x)	**M1**	Horizontal form. Accept $\ln x = \frac{1}{4}(1+\ln x)^2$
	Reduce to $(\ln x)^2 - 2\ln x + 1 = 0$	**A1**	or 3-term equivalent. Condone $\ln x^2$ if later used correctly
	Solve a 3-term quadratic in ln x for x	**M1**	Must see working if solving incorrect quadratic
	Obtain answer x = e	**A1**	Accept e^1
	Obtain answer $y = \frac{1}{2}$ e	**A1**	Exact only with no decimals seen before the exact value. Accept $\frac{e^1}{2}$ but not $\frac{e}{1+\ln e}$
		7	

Question	Answer	Marks	Guidance
5(i)	State answer $-1-\sqrt{3}i$	**B1**	If $-\frac{1}{2}$ given as well at this point, still just B1
		1	

Question	Answer	Marks	Guidance
5(ii)	Substitute $x = -1+\sqrt{3}\mathrm{i}$ in the equation and attempt expansions of x^2 and x^3	**M1**	Need to see sufficient working to be convinced that a calculator has not been used.
	Use $\mathrm{i}^2 = -1$ correctly at least once	**M1**	Allow for relevant use at any point in the solution
	Obtain k = 2	**A1**	
	Carry out a complete method for finding a quadratic factor with zeros $-1+\sqrt{3}\mathrm{i}$ and $-1-\sqrt{3}\mathrm{i}$	**M1**	Could use factor theorem from this point. Need to see working. M1 for correct testing of correct root or allow M1 for three unsuccessful valid attempts.
	Obtain $x^2 + 2x + 4$	**A1**	Using factor theorem, obtain $\mathrm{f}\left(-\frac{1}{2}\right) = 0$
	Obtain root $x = -\frac{1}{2}$, or equivalent, *via* division or inspection	**A1**	Final answer

Question	Answer	Marks	Guidance
5(ii)	**Alternative method 1**		
	Carry out a complete method for finding a quadratic factor with zeros $-1+\sqrt{3}\mathrm{i}$ and $-1-\sqrt{3}\mathrm{i}$ (multiplying two linear factors or using sum and product of roots)	**M1**	Need to see sufficient working to be convinced that a calculator has not been used.
	Use $\mathrm{i}^2=-1$ correctly at least once	**M1**	Allow for relevant use at any point in the solution
	Obtain x^2+2x+4	**A1**	Allow M1A0 for x^2+2x+3
	Obtain linear factor $kx+1$ and compare coefficients of x or x^2 and solve for k	**M1**	Can find the factor by inspection or by long division Must get to zero remainder
	Obtain $k = 2$	**A1**	
	Obtain root $x = -\dfrac{1}{2}$	**A1**	Final answer
			Note: Verification that $x = -\dfrac{1}{2}$ is a root is worth no marks without a clear demonstration of how the root was obtained

Question	Answer	Marks	Guidance
5(ii)	**Alternative method 2**		
	Use equation for sum of roots of cubic and use equation for product of roots of cubic	**M1**	
	Use $\mathrm{i}^2 = -1$ correctly at least once	**M1**	Allow for relevant use at any point in the solution
	Obtain $-\frac{5}{k} = -2 + \gamma$, $\quad -\frac{4}{k} = 4\gamma$	**A1**	
	Solve simultaneous equations for k and γ	**M1**	
	Obtain $k = 2$	**A1**	
	Obtain root $\gamma = -\frac{1}{2}$	**A1**	Final answer
		6	

Question	Answer	Marks	Guidance
6(i)	Correct use of trigonometry to obtain $AB = 2r\cos x$	**B1**	**AG**
		1	

Question	Answer	Marks	Guidance
6(ii)	Use correct method for finding the area of the sector and the semicircle and form an equation in x	**M1**	$\frac{1}{2}\times\frac{1}{2}\pi r^2=\frac{1}{2}(2r\cos x)^2\,2x$
	Obtain $x=\cos^{-1}\sqrt{\frac{\pi}{16x}}$ correctly **AG**	**A1**	Via correct simplification e.g. from $\cos^2 x=\frac{\pi}{16x}$
		2	
6(iii)	Calculate values of a relevant expression or pair of expressions at x = 1 and x = 1.5 Must be working in radians	**M1**	e.g. $x=1$ $\quad 1\to 1.11$ $\quad$ Accept $\mathrm{f}(1)=1.11$ $x=1.5$ $\quad 1.5\to 1.20$ $\quad \mathrm{f}(1.5)=1.20$ $\mathrm{f}(x)=x-\cos^{-1}\sqrt{\frac{\pi}{16x}}$: $\mathrm{f}(1)=-0.111.,\ \mathrm{f}(1.5)=0.3..$ $\mathrm{f}(x)=\cos x-\sqrt{\frac{\pi}{16x}}$: $\mathrm{f}(1)=0.097.,\ \mathrm{f}(1.5)=-0.291.$ For $16x\cos^2 x-\pi$ $\ \mathrm{f}(1)=1.529..,\ \mathrm{f}(1.5)=-3.02..$ Must find values. M1 if at least one value correct
	Correct values and complete the argument correctly	**A1**	
		2	

Question	Answer	Marks	Guidance
6(iv)	Use $x_{n+1} = \cos^{-1}\sqrt{\left(\frac{\pi}{16x_n}\right)}$ correctly at least twice Must be working in radians	**M1**	$1, 1.11173, 1.13707, 1.14225, 1.14329, 1.14349,$ $1.14354, 1.14354$ $1.25, 1.16328, 1.14742, 1.14432, 1.14370$ $1.5, 1.20060, 1.15447, 1.14570, 1.14397, 1.14363$
	Obtain final answer 1.144	**A1**	
	Show sufficient iterations to at least 5 d.p. to justify 1.144 to 3 d.p. or show there is a sign change in the interval (1.1435, 1.1445)	**A1**	
		3	

Question	Answer	Marks	Guidance
7(i)	Separate variables correctly and attempt integration of at least one side	**B1**	$\int e^{-y}\,dy = \int xe^{x}\,dx$
	Obtain term $-e^{-y}$	**B1**	B0B1 is possible
	Commence integration by parts and reach $xe^{x} \pm \int e^{x}\,dx$	**M1**	B0B0M1A1 is possible
	Obtain $xe^{x} - e^{x}$	**A1**	or equivalent
			B1B1M1A1 is available if there is no constant of integration
	Use $x = 0$, $y = 0$ to evaluate a constant, or as limits in a definite integral, in a solution with terms ae^{-y}, bxe^{x} and ce^{x}, where $abc \neq 0$	**M1**	Must see this step
	Obtain correct solution in any form	**A1**	e.g. $e^{-y} = e^{x} - xe^{x}$
	Rearrange as $y = -\ln(1-x) - x$	**A1**	or equivalent e.g. $y = \ln\dfrac{1}{e^{x}(1-x)}$ ISW
		7	
7(ii)	Justify the given statement	**B1**	e.g. require $1-x>0$ for the ln term to exist, hence $x<1$ Must be considering the range of values of *x*, and must be relevant to *their y* involving $\ln(1-x)$
		1	

Question	Answer	Marks	Guidance
8(i)	State or imply the form $\dfrac{A}{2x+1}+\dfrac{B}{2x+3}+\dfrac{C}{(2x+3)^2}$	**B1**	
	Use a correct method to find a constant	**M1**	
	Obtain the values $A = 1$, $B = -1$, $C = 3$	**A1 A1 A1**	
	[Mark the form $\dfrac{A}{2x+1}+\dfrac{Dx+E}{(2x+3)^2}$, where $A = 1$, $D = -2$ and $E = 0$, B1M1A1A1A1 as above.]		Full marks for the three correct constants – do not actually need to see the partial fractions
		5	
8(ii)	Integrate and obtain terms $\frac{1}{2}\ln(2x+1)-\frac{1}{2}\ln(2x+3)-\dfrac{3}{2(2x+3)}$ [Correct integration of the A, D, E form of fractions gives $\frac{1}{2}\ln(2x+1)+\dfrac{x}{2x+3}-\frac{1}{2}\ln(2x+3)$ if integration by parts is used for the second partial fraction.]	**B1 B1 B1**	**FT** on A, B and C.
	Substitute limits correctly in an integral with terms $a\ln(2x+1)$, $b\ln(2x+3)$ and $c/(2x+3)$, where $abc \neq 0$ If using alternative form: $cx/(2x+3)$	**M1**	value for upper limit – value for lower limit 1 slip in substituting can still score M1 Condone omission of $\ln(1)$
	Obtain the **given answer** following full and correct working	**A1**	Need to see at least one interim step of valid log work. **AG**
		5	

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td>9(i)</td><td>Carry out correct method for finding a vector equation for AB</td><td>M1</td><td></td></tr>
<tr><td></td><td>Obtain $(\mathbf{r}=)\mathbf{i}+2\mathbf{j}-\mathbf{k}+\lambda(2\mathbf{i}-\mathbf{j}+2\mathbf{k})$, or equivalent</td><td>A1</td><td></td></tr>
<tr><td></td><td>Equate two pairs of components of general points on their AB and l and solve for λ or for μ</td><td>M1</td><td>$\begin{pmatrix}1+2\lambda\\2-\lambda\\-1+2\lambda\end{pmatrix}=\begin{pmatrix}2+\mu\\1+\mu\\1+2\mu\end{pmatrix}$</td></tr>
<tr><td></td><td>Obtain correct answer for λ or μ, e.g. $\lambda = 0$, $\mu = -1$</td><td>A1</td><td></td></tr>
<tr><td></td><td>Verify that all three equations are not satisfied and the lines fail to intersect ($\neq$ is sufficient justification e.g. $2 \neq 0$)
Conclusion needs to follow correct values</td><td>A1</td><td>Alternatives
<table>
<tr><th>A</th><th>λ</th><th>μ</th><th></th><th>B</th><th>λ</th><th>μ</th><th></th></tr>
<tr><td>ij</td><td>$2/3$</td><td>$1/3$</td><td>$\frac{1}{3}\neq\frac{5}{3}$</td><td></td><td>$-1/3$</td><td>$1/3$</td><td>$\frac{1}{3}\neq\frac{5}{3}$</td></tr>
<tr><td>ik</td><td>0</td><td>–1</td><td>$2\neq 0$</td><td></td><td>–1</td><td>–1</td><td>$2\neq 0$</td></tr>
<tr><td>jk</td><td>1</td><td>0</td><td>$3\neq 2$</td><td></td><td>0</td><td>0</td><td>$3\neq 2$</td></tr>
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Question	Answer	Marks	Guidance
9(ii)	State or imply midpoint has position vector $2\mathbf{i}+\frac{3}{2}\mathbf{j}$	B1	
	Substitute in $2x - y + 2z = d$ and find d	M1	Correct use of *their* direction for AB and *their* midpoint
	Obtain plane equation $4x - 2y + 4z = 5$	A1	or equivalent e.g. $\mathbf{r}.\begin{pmatrix}2\\-1\\2\end{pmatrix}=\frac{5}{2}$
	Substitute components of l in plane equation and solve for μ	M1	Correct use of their plane equation.
	Obtain $\mu = -\frac{1}{2}$ and position vector $\frac{3}{2}\mathbf{i}+\frac{1}{2}\mathbf{j}$ for the point P	A1	Final answer Accept coordinates in place of position vector
		5	

Question	Answer	Marks	Guidance
10(i)	State correct expansion of $\sin(3x+x)$ or $\sin(3x-x)$	**B1**	B0 If their formula retains ± in the middle
	Substitute expansions in $\frac{1}{2}(\sin 4x + \sin 2x)$	**M1**	
	Obtain $\sin 3x \cos x = \frac{1}{2}(\sin 4x + \sin 2x)$ correctly	**A1**	Must see the $\sin 4x$ and $\sin 2x$ or reference to LHS and RHS for A1 **AG**
		3	
10(ii)	Integrate and obtain $-\frac{1}{8}\cos 4x - \frac{1}{4}\cos 2x$	**B1 B1**	
	Substitute limits $x = 0$ and $x = \frac{1}{3}\pi$ correctly	**M1**	In their expression
	Obtain answer $\frac{9}{16}$	**A1**	From correct working seen.
		4	

Question	Answer	Marks	Guidance
10(iii)	State correct derivative $2\cos 4x + \cos 2x$	**B1**	
	Using correct double angle formula, express derivative in terms of $\cos 2x$ and equate the result to zero	**M1**	
	Obtain $4\cos^2 2x + \cos 2x - 2 = 0$	**A1**	
	Solve for x or $2x$ (could be labelled x) $\left(\cos 2x = \dfrac{-1 \pm \sqrt{33}}{8}\right)$	**M1**	Must see working if solving an incorrect quadratic The roots of the correct quadratic are -0.843 and 0.593 Need to get as far as $x = \ldots$ The wrong value of x is 0.468 and can imply M1 if correct quadratic seen Could be working from a quartic in $\cos x$: $16\cos^4 x - 14\cos^2 x + 1 = 0$
	Obtain answer x = 1.29 only	**A1**	
		5	